Best Paper Award

Topology Preserving Maps From Virtual Coordinates for Wireless Sensor Networks

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***Abstract*—A method of obtaining a topology preserving map from virtual coordinates of a sensor network is presented. In a Virtual Coordinate System (VCS), a node is identified by a vector containing its distances, in hops, to a set of nodes called anchors. VCS is a higher dimensional abstraction of the connectivity map of nodes, with dimensionality defined by the number of anchors. Physical layout information such as physical voids and even relative physical positions of sensor nodes with respect to X-Y directions are absent in a VCS description, and obtaining the physical topology has not been possible up to now. A novel technique, based on Singular Value Decomposition, is presented to extract a topology preserving map from VCS. Three options with different computation and communication complexities, as a result of using different subsets of coordinates as the input, are presented and analyzed; the input for the three cases consist of a) the entire virtual coordinate set, b)only the virtual coordinates of anchors, and c) virtual coordinates of a random set of nodes. Evaluation results indicate that last two approaches achieve comparable accuracy to the first, but with significantly less complexity. Topology preserving maps for networks representing a variety of topologies and shapes are extracted. A new metric termed Topology Preservation Error (E_{TP}) is defined to evaluate the topology preservation; it accounts for both the number of node flips and degree of the flips. The techniques extract topology preserving maps with E_{TP} less than 2%.**

Keywords- Topology-Preserving Map, Virtual Coordinates, Localization, Routing, Singular Value Decomposition, Wireless Sensor Networks

I. INTRODUCTION

Virtual Coordinate Routing (VCR) and Geographical Routing (GR) are two main classes of address-based routing schemes for Wireless Sensor Networks (WSNs). Geographical routing relies on location information of nodes, which may be obtained by a GPS like mechanism or a localization algorithm. Routing is carried out by using directional information that can be derived from individual node locations. Infeasibility and cost of GPS, localization errors, and poor routability under concave physical voids are some of its disadvantages. VCR is based on a Virtual Coordinate System (VCS) that characterizes each node by a coordinate vector of size M, consisting of the shortest hop distance to each of a set of M anchors [4][5][14]. This connectivity based higher dimensional representation of WSN has some attractive properties such as routing without requiring geographical information [4-7][14] that makes it useful for resource-starved WSNs. Here, the number of anchors becomes networks' dimensionality, and the network's connectivity information is embedded in Virtual Coordinates (VCs); thus physical voids are transparent in Virtual Space (VS).But VCs lose the directional information of nodes' positions. Physical domain to virtual domain transformation is one-to-many as VCs are insensitive to directions. If a sufficient number of anchors are not properly deployed, the network will suffer from identical coordinates and local minima problem [6] resulting in logical voids. Identification of the optimal number of anchors and proper anchor placement remains a major challenge [6].

Many of the problems associated with VCS are due to the lack of information about physical network topology. Physical layout information such as physical voids, relative physical positions of sensor nodes with respect to X-Y directions, and even explicit connectivity information are absent in a VCS description. These difficulties can be overcome if information on physical topology is available. Combining VC information and position or direction information in one network essentially would combine the advantages of VCR and GR schemes. However, this has to be achieved without inheriting the disadvantages associated with obtaining location information or localization. Obtaining topological and directional information from VCS, i.e., VCS to physical coordinate transformation is challenging. Obtaining the physical topology of a network from the set of VCs has not been possible up to now. In this paper, we present the theoretical basis and techniques to obtain a topology map that preserves the 2-D physical topology of a sensor network, including the geographical voids and relative Cartesian directional information.

The closest theoretical result to our work is MultiDimensional Scaling (MDS) [16], used for the analysis of proximity data to reveal the structure underlying the data. It starts with the distance matrix D,containing all the pair wise distances between network nodes.$D = [d_{ij}]_{N\times N}$, where, N is the number of nodes in the network and d_{ij} is the distance from node i to node j with $d_{ij} = d_{ji}, \geq 0$ and $d_{jj} = 0$. In general d_{ij} can be any distance metric, but there is a possibility for the algorithm to fail if d_{ij} is not the Euclidean distance. Generating D based on hop distances requires all the nodes to serve as anchors, an extremely expensive proposition that calculates and stores information about the distances between each pair of nodes. If such information were available at each node, 100% routing can be achieved even without the topology map. MDS is therefore not practical or applicable for generating topology preserving maps of WSN. Our novel method, based on Singular Value Decomposition (SVD), generates a two dimensional topology map using a set of M anchors, where $M << N, N$ being the number of nodes.

Topology preserving maps preserve the neighborhood information but they have neither the exact coordinates nor the relative distances compared to the original topology. These maps are rotations, and/or translations from the real topology of the network. With the information of geographical locations, topology map can be used for localization as well. WSN routing [1], boundary node identification for proper anchor placements [4][5][14], backbone identification, and identification of geographic voids, are among the examples that can significantly benefit from a topology preserving map. Topology preserving maps provide the ability to determine and visualize the structural characteristics of a WSN. Visualizing network infrastructure using VCS of the network, which defines the node locations in a higher dimensional space is extremely difficult except for trivial network topologies. However, a topology preserving map, such as that we are able to derive based on the proposed technique, would provide a physical representation. It will facilitate network design and network management processes by exposing potential weaknesses in the infrastructure and opportunities to improve its robustness and performance.

This paper presents techniques for obtaining topology preserving maps of networks from VCS. Results presented demonstrate that it is able to provide directional relationships among nodes, and identify features such as physical voids and network boundaries. Even though we focus on WSN context here, the proposed technique is applicable to a broader class of networks. After presenting the theoretical foundation for obtaining the topology preserving map using the set of VCs, we refine the method to reduce complexity. We demonstrate that we can achieve almost as accurate topology maps by using only the coordinates of the anchor nodes, or those of a set or random nodes, thus significantly reducing the computational and communication complexity. A novel performance evaluation metric is proposed to capture the percentage of node position flips due to the transformation, and issued for evaluating the accuracy of the resulting topology preserving maps.

Section II reviews the background, and Section III discusses mapping nodes to Cartesian coordinates and thus a topology preserving map. Section IV presents a performance evaluation metric for topology maps and Section V presents the results. Section VI addresses implementation issues. Section VII presents conclusions and future work.

II. Background

A. Geographic Routing (GR) and Virtual Coordinate Routing(VCR)

In Geographic Routing, the physical location of nodes is used for addressing of the node as well as for routing. Greedy Perimeter Stateless Routing (GPSR) [9] makes greedy forwarding decisions until it fails, for example due to a geographical void, and attempts to recover by routing around the perimeter of the void. The Greedy Other Adaptive Face Routing (GOAFR) [11], is a geometric ad-hoc algorithm combining greedy forwarding and face routing to overcome the local minima issue. Greedy Path Vector Face Routing with Path Vector Exchange GPVFR/PVEX [12] is similar to [11] but it uses more information about the planar graph.

VCR examples include an algorithm which uses all the perimeter nodes as anchors[14].When a message reaches a local minimum, an expanding ring search is performed till a closer node is found or TTL expires. In Virtual Coordinate assignment protocol (VCap), the coordinates are defined based on hop distances [4]. At local minima, VCR causes a packet to follow a rule called local detour. In Logical Coordinate based Routing (LCR) [5] backtracking is used when greedy forwarding fails at a local minimum. Aligned virtual coordinate system (AVCS) [13] reevaluates VCs by averaging its own coordinate with neighboring coordinates to overcome local minima. In Axis-based Virtual Coordinate Assignment Protocol (ABVCap) [18], each node is assigned a 5-tuple VCs corresponding to longitude, latitude, ripple, up, and down. All the VCR protocols rely mainly on Greedy forwarding, followed by a backtracking scheme to overcome the local minima issue.

B. Localization [2][19]

Here we focus on relative localization techniques, as global localization is realizable through relative localization and the actual positions of a subset of nodes or anchors. Centralized and distributed algorithms are available for relative localization.

Distributed algorithms use Received Signal Strength Indication (RSSI), Radio Hop Count, Time Difference of Arrival and Angle of arrival for relative localization. RSSI uses signal strength to estimate the distance between nodes while. Radio Hop Count uses hop distance. Latter uses a probabilistic correction equation to approximate the hop distance to real distance [2].

Centralized algorithm has its own advantages [2]. Two main centralized algorithms are SemiDefinite Programming (SDP) and MDS-MAP. Former algorithm develops geometric constraints between nodes and represented as linear matrix inequalities (LMIs) then simply solve for the intersection of the constraints. Unfortunately, not all geometric constraints can be expressed as LMIs, which preclude the algorithm's use in practice. MDS-MAP is Multidimensional scaling based on pair-wise distances measured using RSSI [2].

C. Localization Vs Topology Preserving Maps

Topology preserving maps discussed in this paper deviates from the localization. The relative localization schemes expect the relative distances to be accurate. Thus given the absolute position of a subset of nodes, global localization is realizable. In contrast, in topology maps what is important is the topology preservation, not the node distances. The derived topology should be homeomorphic (topologically isomorphic) to the physical layout of the sensor network, i.e., between two topological spaces there has to be a continuous inverse function. In our case, a mapping preserves the topological properties of the physical network topology. However, this does not preclude the transformation of the topology preservation map to obtain node localization.

III. Cartesian Coordinates and Topology maps

This section presents a novel technique for obtaining a Cartesian coordinate based map of a sensor network from its VC set. The objective is to characterize each node with a

(x, y) coordinate pair, resulting in a topology preserving map that is homeomorphic to the networks physical layout, and preserves information about node connectivity, physical layout and physical voids.

Subsection A develops the technique by starting with the VCs of all the nodes to obtain the Cartesian coordinates of the nodes, which collectively result in a topology preserving map. In Subsection B, we present a significantly more efficient version of the technique that uses information of only a small subset of nodes to evaluate the transformation matrix. The technique is illustrated using 1) the set of anchor nodes, and 2) a random set of nodes, as the selected subset. Finally, Subsection C proposes a method of calculating node's Cartesian coordinate with lower computational complexity.

A. 2-D topology preserving map from VCs

Consider a sensor network with N nodes and M anchors. Thus each node is characterized by M-length VC vector. Let $\underline{P}$ be the $N \times M$ matrix containing the VCs of all the nodes, i.e., the i^{th} row corresponds to the M- length VC vector of the i^{th} node, and j^{th} column corresponds to the virtual ordinate of all the nodes in the network with respect to j^{th} anchor. Since only a small set of nodes are anchors, $N >> M$. Denote the Singular Value Decomposition (SVD) [10] of $\underline{P}$ as

$$\underline{P} = \underline{U}.\underline{S}.\underline{V}^T \quad (1)$$

where, $\underline{U}, \underline{S}$ and $\underline{V}$ are $N \times N$, $N \times M$, and $M \times M$ matrices respectively. $\underline{U}$ and $\underline{V}$ are unitary matrices, i.e., $\underline{U}^T\underline{U} = I_{N\times N}$ and $\underline{V}^T\underline{V} = I_{M\times M}$. SVD extracts and packages the salient characteristics of the dataset $\underline{P}$ providing an optimal basis for $\underline{P}$. Moreover $\underline{V}$ is an optimal basis of $\underline{P}^T$ .i.e. $\underline{V}$ spans $\mathcal{R}^M$. Then, $\underline{U}.\underline{S}$ gives the coordinates for the data $\underline{P}$ under the new basis $\underline{V}$. Since $\underline{S}$ is a diagonal matrix, where diagonal elements are the singular values arranged in their descending order, elements in $\underline{S}$ provides unequal weights on columns of $\underline{U}$.Therefore,

$$\underline{P}_{SVD} = \underline{U} \cdot \underline{S} \quad (2)$$

$\underline{P}_{SVD}$ is a $N \times M$ matrix that describes each node with a new set of M-length coordinate vectors. Using the unitary property of $\underline{V}$, it can be seen that it is the projection of $\underline{P}$ on to $\underline{V}$, i.e.,

$$\underline{P}_{SVD} = \underline{P} \cdot \underline{V} \quad (3)$$

The columns of P_{SVD} are arranged in the descending order of information about the original coordinate set. With many SVD based analyses, the first component contains the most important and discriminating information and is indispensable. However, as we show below, the second and third components are more important for the generation of topology preserving maps.

First seven ordinates given by Eq. (3) are plotted against the corresponding physical positions of the nodes for the networks shown in Fig. 1 (a) and 2 (a). As we observed (See Fig. 1 and 2) initial triplet of SVD coordinates are the dominating coordinates while the rest are similar to Fourier basis vectors with less significant amplitudes.

The set of VCs have the connectivity information embedded in it though it loses directional information. All the nodes that are h hops away from the j^{th} anchor have h as the j^{th} ordinate. Every ordinate propagates as concentric circles

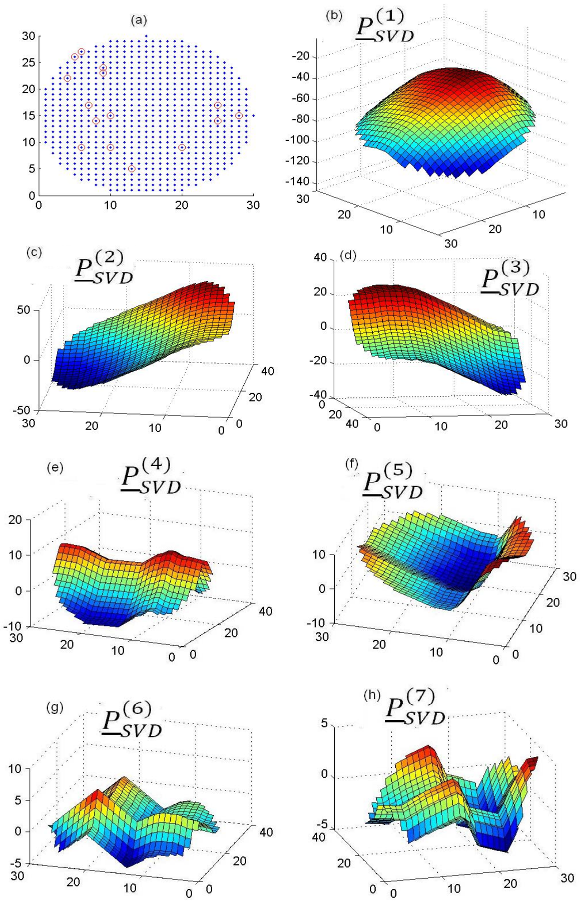

Figure 1.a) Circular network of 707 nodes and b)-h) First eight SVD coordinate vectors plotted against the physical positions.

centered at the corresponding anchor, while the angular information is completely lost. Thus, we can expect the most significant ordinate based on SVD, i.e., first column of $\underline{P}_{SVD}$ to contain radial information, which will not give us enough information to identify nodes distinctly (See Fig. 1 (b)) and Fig. 2 (b)). As SVD provides an orthonormal basis, 2nd and 3rd ordinates are orthogonal to 1st ordinate while being perpendicular to each other as illustrated in Fig. 3. This leads to the fact that the second and third columns of $\underline{P}_{SVD}$ provide a set of 2-dimensional Cartesian coordinates for node positions. Thus instead of the M coordinates of a row of $\underline{P}_{SVD}$ to characterize a node, the second and third columns can be used as Cartesian coordinates, i.e.,

$$[X_{SVD}, Y_{SVD}] = [\underline{P}_{SVD}{}^{(2)}, \underline{P}_{SVD}{}^{(3)}] \quad (4)$$

where $\underline{P}_{SVD}{}^{(j)}$,(j=2,3), is the j^{th} column of $\underline{P}_{SVD}$ and $[X_{SVD}, Y_{SVD}]$ is the Cartesian coordinate matrix of the entire node set, i.e., its i^{th} row, $[x_{SVD,i}, y_{SVD,i}]$, is considered to be the Cartesian coordinates of i^{th} node, Therefore by construction of SVD coordinates, we assert that the second and third basis vectors form an orthogonal Cartesian plane for the network and corresponding ordinates gives us approximated Cartesian coordinates (Eq. (4)). Most fascinating is the fact that these Cartesian coordinates are estimated without having any kind of physical directional or positioning information beyond the radial information (hop distance) with respect to the anchors.

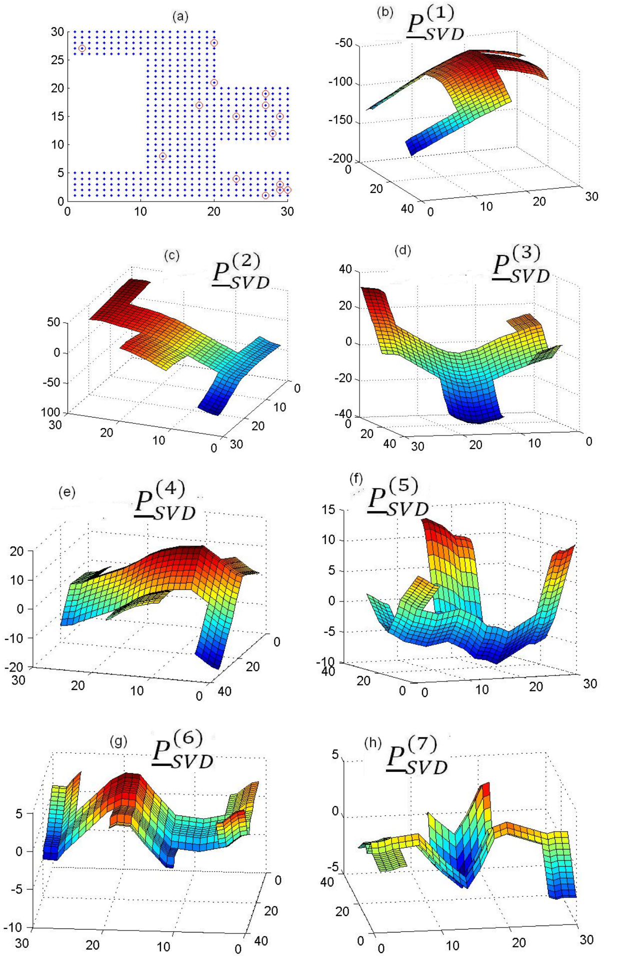

Figure 2.b) Odd shaped network with 550 nodes, and b)-h) First eight SVD coordinate vectors plotted against the physical positions.

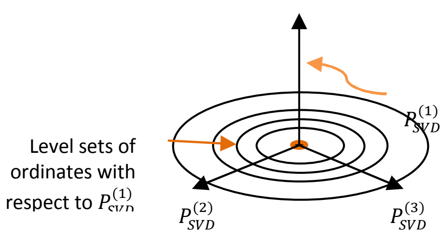

Figure 3.Three main directions from SVD.

The topology preserving map thus obtained reflects the original topological characteristics of the network. One can even identify features such as physical voids that were not apparent in the VC based description.

B. Generation of Cartesian coordinate set using VCs of a subset of nodes

Cartesian coordinates can be obtained by multiplying the node's VC by $\underline{V}$ as in Eq. (3) and (4). $\underline{V}$ is evaluated based on $\underline{P}$ which is a $N \times M$ matrix that consists of VCs of entire network. With sensor networks, it is crucial to reduce communication and computation overheads involved. This section presents a process to generate the transformation matrix $\underline{V}$ with much smaller subset of data of $\underline{P}$, thus significantly reducing the computation overhead.

Let $\underline{Q}$ be a subset of the node coordinate set. Then we can write the SVD of $\underline{Q}$as,

$$\underline{Q} = \underline{U_Q} \cdot \underline{S_Q} \cdot \underline{V_Q}^T \tag{5}$$

$\underline{V_Q}$ is a basis for $\mathcal{R}^M$. Similarly $\underline{V}$ is also a basis of $\mathcal{R}^M$. Therefore, we can write

$$\underline{V} = \underline{V_Q} \cdot A \tag{6}$$

Where A is a rotation matrix.

We consider two options for selecting the subset of nodes for computing $\underline{V_Q}$ is as described below.

1) Use the set of coordinates of the M anchor nodes ($\underline{Q}$=$\underline{Q_M}$)

Then, $\underline{U_Q}$, $\underline{S_Q}$ and $\underline{V_Q}$ of Eq. (5) are $M \times M$ matrices, where $\underline{U_Q}$ and $\underline{V_Q}$ are unitary matrices. Note that $\underline{V_Q}$ has the same size as $\underline{V}$ in Eq. (1). Following the same procedure as earlier

$$\underline{\tilde{P}_{SVD}} = \underline{P} \cdot \underline{V_Q} \tag{7}$$

The approximated Cartesian coordinates can be written as

$$[\tilde{X}_{SVD}, \tilde{Y}_{SVD}] = [\underline{\tilde{P}_{SVD}}^{(2)}, \underline{\tilde{P}_{SVD}}^{(3)}] \tag{8}$$

$[\tilde{X}_{SVD}, \tilde{Y}_{SVD}]$is the new Cartesian coordinate set of the network where i^{th} row of $[\tilde{X}_{SVD}, \tilde{Y}_{SVD}]$ is the Cartesian coordinate of i^{th} node.

2) Use a set of R randomly selected nodes ($\underline{Q} = \underline{Q_R}$)

For R nodes $\underline{\mathrm{Q_R}}$ is $\mathrm{R} \times \mathrm{M}$, where M is the number of anchors. SVD of $\underline{\mathrm{Q_R}}$ is evaluated as similar to Eq. (5). Eq. (7) and (8) are valid except sizes of $\underline{U_Q}$ and $\underline{S_Q}$ are $R \times R$ and $R \times M$ respectively.

In sum, $\underline{Q_M}$ and $\underline{Q_R}$ are thus $M \times M$ and $R \times M$ matrices, containing subsets of M and R rows of $\underline{P}$ respectively, with $N >> M, R$.

C. An alternative computationally efficient approach

Computational power and memory available at a node is limited. Conventional SVD calculation of $\underline{P_{N\times M}}, (N \gg M)$, which involves computing $\underline{U}$, $\underline{S}$ and $\underline{V}$, has approximately $(4N^2M + 8NM^2 + 9M^3)$ operations [8]. In addition, the memory requirement is approximately the sizes of $\underline{V}$,$\underline{S}$ and $\underline{P}$ that is $(M \times M + N \times N + N \times M)$. In this section, we present a technique for enhancing the efficiency of the computation. Note that $\underline{U}$ is a byproduct of SVD, and is not necessary for topology map computation. From Eq. (1), (3) and (4),

$$\underline{P}_{SVD,(i,j)} = \left(\underline{P}^T\right)^{(i)} \cdot \underline{V}^{(j)} \tag{9}$$

$\underline{P}_{SVD,(i,j)}$ is the $(i,j)^{\text{th}}$ element of $\underline{P}_{SVD}$, $\left(\underline{P}^T\right)^{(i)}$is the i^{th} row of $\underline{P}$. This can be interpreted as the coordinate vector of the node i. Also $\underline{V}^{(j)}$ is the j^{th} basis vector/column of $\underline{V}$.

$$[x_{SVD,i}, y_{SVD,i}] = [(coordinates\ of\ node\ i) \cdot \underline{V}^{(2)},\ (coordinates\ of\ node\ i) \cdot \underline{V}^{(3)}]$$

Thus, $\underline{V}^{(2)}$ and $\underline{V}^{(3)}$ are sufficient to evaluate Cartesian coordinate of $[x_{SVD,i}, y_{SVD,i}]$ of node i. Define $\underline{C}$ as

$$\underline{C} = \underline{P}^T.\underline{P} = \underline{V}.\underline{S}^2.\underline{V}^T$$
$$\underline{CV} = \underline{V}.\underline{S}^2 \tag{10}$$

$\underline{C}$ is a $M \times M$ matrix. This is an eigenvalue problem [10]. There for let us solve $\underline{C}\ v = \lambda v$, where v is an eigenvector of $\underline{C}$ that is a column of $\underline{V}$. Eigen values can be found by solving

$$\left|\underline{C} - \lambda I\right| = 0 \tag{11}$$

λs are the eigenvalues. The eigenvectors corresponding to second and third *largest* eigenvalues provides the second and third columns of $\underline{V}$. Now $\left[x_{SVD,i}, y_{SVD,i}\right]$ can be evaluated locally without calculating entire $\underline{V}$. Also $\underline{U}_{N\times N}$ is not evaluated at all, which reduces the memory consumption significantly. Therefore, the memory consumption is upper bounded by $(M \times M) + (N \times M)$. If $\underline{C} = \underline{Q}^T.\underline{Q}$, then memory consumption is upper bounded by $2(M \times \underline{M})$.

Number of computations required in the proposed method of calculating $\underline{V}$ is upper bounded by $(4M^2N + 8M^3)$ [8], which is the computations associated with calculation of entire $\underline{V}$ and $\underline{S}$. As $N \gg M$ for the WSN applications, this method is significantly less complex compared to full SVD implementation (See Table I). For example, if the number of anchors in the network is set to $M \leq 0.01N$, the upper bound of computations with this method is only 0.99% of the computations required for a full SVD based calculation with Eq. (3) and (4) ,indicating a significant complexity reduction.

TABLE I. COMUTATIONAL COMPLEXITY AND MEMORY USAGE COMPARISION

Method	Proposed method of estimating $\underline{V}$	Full SVD implementation
# Computations N ≫M	*Upper bounded by* $(4M^2N + 8M^3)$[8]	$(4N^2M + 8NM^2 + 9M^3)$[8]
Memory usage	*Upper bounded by* $(M \times M) + (N \times M)$	$(M \times M) + (N \times N) + (N \times M)$

IV. A METRIC FOR EVALUATING TOPOLOGY PRESERVATION

Evaluating the accuracy of topology preservation of the sensor node map generated using estimated Cartesian coordinates (See Eq. (4)) is essential for investigating the effectiveness of the proposed scheme. While visual inspection can provide preliminary evidence of its effectiveness, a formal metric is needed for quantifying the accuracy. A quantitative parameter to express the error provides a framework to compare and improve different mapping techniques. An effective metric should be able to capture and quantify the failures to preserve the topology of the real node map and the neighborhoods. Here we develop a metric that can be used for this purpose.

A method based on coloring of nodes is used in [15] to show whether a neighborhood has been altered in the topology map. In [15] and [17], error is quantified as the difference of the positions in the real map and the topology map, and the residual variance, respectively. The focus of our paper is topology preserving maps based on hop distances. The requirement is that the map from calculated $[X_{SVD}, Y_{SVD}]$ set is homeomorphic to the physical layout, and preserves information about node connectivity, physical layout and physical voids. Thus the actual physical distance is not of significance, and the metrics in [15] and [17] are not appropriate.

Consider as an example, a 1-D network with 6 nodes numbered 1 to 6 as in Fig. 4(a). Figs 4(b) and (c) show two derived maps that need to be evaluated. If all the nodes are in same order as in initial topology then neighborhood preservation error has to be 0%. Node 3 in Fig. 4 (b) has flipped two node positions. The error metric should identify the number of out of order nodes as well as the degree of the error/node flips (one node and two node positions respectively for Fig. 4 (b)).

Consider a 1-D network with N_k nodes and define an indicator function $I_{i,j}$ where

$$I_{i,j} = \begin{cases} 1 & i \text{ and } j \text{ are out of order compared to original placement} \\ 0 & i \text{ and } j \text{are in same order as original placement or } i = j \end{cases}$$

$$i,j = 1\ to N_k$$

Then, Number of out of order pairs = $\sum_{\text{all i,j}} (\mathrm{I_{i,j}})$

The total number of possible pairs in an N_k node network is $\mathrm{P}_2^{N_k}$. We define the following metric:

$$\text{Topology Preservation Error } = \mathrm{E_{TP}} = \frac{\sum_{\text{all i,j}}(\mathrm{I_{i,j}})}{\mathrm{P_2^N}} \tag{12}$$

For the network in Fig. 4 (b), N =6 and:

$$\mathrm{E_{TP}} = (\mathrm{I_{3,4} + I_{3,5} + I_{4,3} + I_{5,3}})/\mathrm{P}_2^6 \times 100\%$$
$$= (\mathrm{I_{3,4} + I_{3,5}})/\mathrm{C}_2^6 \times 100\% = 13.3\%$$

Node 1 and 2 are in right position compared to the rest while node 3 is shifted by 2 positions. Moreover, nodes 4 and 5 flipped their positions by 1. Therefore total node flips are 4 and $\mathrm{E_{TP}}$ is 13.3%

A topology preserving map is invariant to rotations. Thus, for Fig.4 (c), where nodes are just reversed, $\mathrm{E_{TP}}$ should be zero.To handle such cases, the two lines being compared need to be adjusted for any rotations.

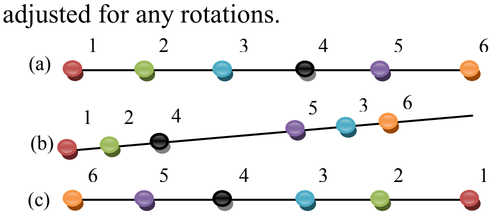

Figure 4. Original layout (a) and two example topology maps (b), (c)

To extend the 1-D equation to 2-D topologies, we evaluate the 2-D topology by considering all contiguous line segments in two orthogonal directions (say $\vec{H}$ and $\vec{V}$) of the physical map. Let us assume there are H lines in $\vec{H}$direction and V lines in $\vec{V}$direction in the network, then,

Topology Preservation Error in $\vec{H}$ direction % =

$$\mathrm{E_{TP|\vec{H}}} = \frac{\sum_{\mathrm{H}}\sum_{\text{all i,j}}(\mathrm{I_{i,j}})}{\sum_{\mathrm{H}}\mathrm{P_2^{N_h}}} \tag{13}$$

$i, j$are nodes in each horizontal line and each line has N_h nodes. Similarly, error in vertical direction is evaluated as

Topology Preservation Error in $\vec{V}$ direction % =

$$\mathrm{E_{TP|\vec{V}}} = \frac{\sum_{\mathrm{V}}\sum_{\text{all i,j}}(\mathrm{I_{i,j}})}{\sum_{\mathrm{V}}\mathrm{P_2^{N_v}}} \tag{14}$$

i, j are nodes in each vertical line and each has N_v nodes. The overall Topology Preservation Error can be defined as:

$$\mathrm{E_{TP}} = \frac{\sum_{\mathrm{V}}\sum_{\text{all i,j}}(\mathrm{I_{i,j}}) + \sum_{\mathrm{H}}\sum_{\text{all i,j}}(\mathrm{I_{i,j}})}{\sum_{\mathrm{H}}\mathrm{P_2^{N_h}} + \sum_{\mathrm{V}}\mathrm{P_2^{N_v}}} \tag{15}$$

V. RESULTS

The performance of the proposed methods are evaluated next using four examples representative of a variety of networks: a uniformly distributed circular network with 707 nodes (Fig. 5), an odd shaped network with 550 nodes (Fig. 6), a 496-node circular shaped network with three physical voids/holes (Fig. 7), and a network of 343 nodes mounted on walls of a building (Fig. 8). Figure identified as (a) in Fig. 5-8 show the physical maps of the four cases respectively. Communication range of a node in all four networks is unity. MATLAB® 2009b was used for the computations. Topology maps are generated based on methods summarized in Table II.

Unless otherwise indicated, the results shown correspond to fifteen randomly placed anchors in each of the networks. Fig. 5-8 (b) is the topology preserving map constructed based on Eq. (4) using entire VC set of each network. Therefore Networks in Fig. 5-8(b) uses input data matrices of sizes 707x15, 550x15, 496x15 and 343x3 respectively (Case 1,Table II). Then Fig. 5-8 (c) are the topology maps created using anchors' coordinate set, that is using Eq. (7) and (8) based on the input data matrices of size 15x15, 15x15, 15x15 and 3x 3 respectively (Case 2,Table II). Topology maps in Fig. 5-8 (d) are created on coordinates of 10 randomly selected nodes of sizes 10x15, 10x15, 10x15 and 10x 3(Case 3,Table II).Finally, Fig 5-8 (e) assumes all the nodes in the network are anchors and generate VCSs of sizes 707x707, 550x550, 496x496 and 343x343for corresponding network. Then use that VC set for topology map generation (Case 4, Table II). Case 3 is more efficient in terms of memory consumption and computational complexity.

TABLE II. FOUR DIFFERENT TOPOLOGY MAP GENERATION APPROACHES FOR WSNS OF N NODES AND M ANCHORS

Case	Description (Eq. (4))	Input data matrix ($\underline{P}$)size
1	$[X_{SVD}, Y_{SVD}]$ is generated based on entire VC set	$N \times M$
2	$[X_{SVD}, Y_{SVD}]$ is generated based on anchors' VC set	$M \times M$
3	$[X_{SVD}, Y_{SVD}]$ is generated based on randomly selected nodes' VC set	$R \times M$
4	$[X_{SVD}, Y_{SVD}]$ is generated based on VC set when all the nodes are anchors	$N \times N$

A key observation we can draw from Fig. 5-7 is that the constructed topology maps are nonlinearly scaled and rotated compared to the actual network map. Yet, the original and constructed maps are topologically isomorphic. Starting just with VCs, without explicit knowledge of geographical information, the generated topology maps have captured significant features such as the physical voids of the original network.

In contrast to previous cases, the topology maps of Fig. 8 (b), (c), (d) are simply a rotated and linear scaled version of the original. In this network, we used only three anchors. Moreover, from Fig. 8 topology maps we can draw a valuable conclusion that we can significantly reduce the number of anchors required for topology map generation with good anchor placement. It is topology preserving to a very high degree as intended. The closeness in shape of Fig. 8(b)-(d) to the original however indicate that an appropriately placed

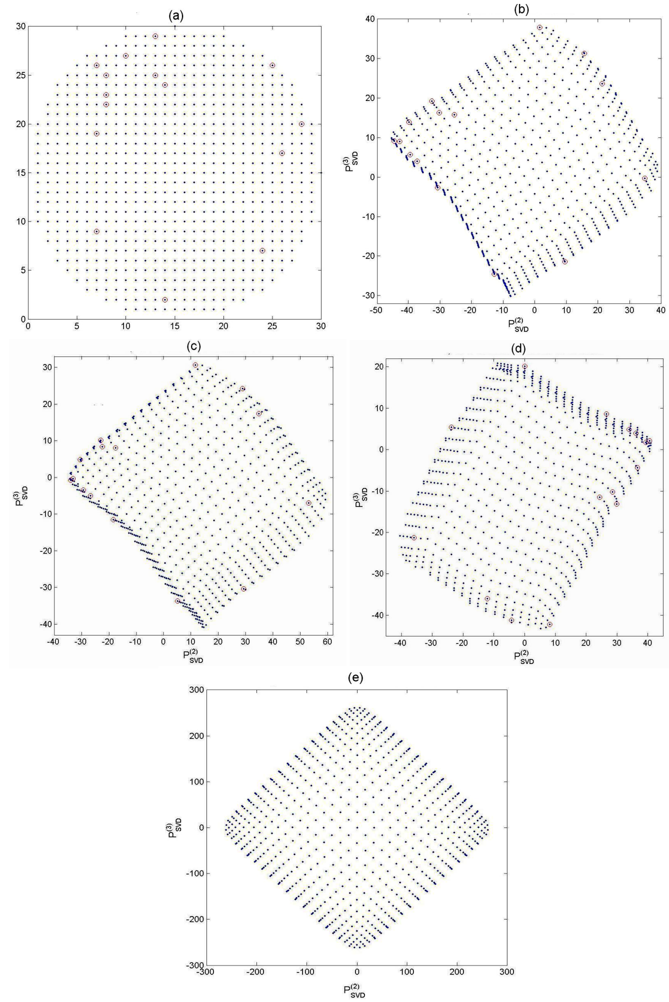

Figure 5.(a) Circular network,$[X_{SVD}, Y_{SVD}]$ is generated based on (b) Case 1:entire VC set,(c) Case 2: anchors' coordinate set(d) Case 3:randomly selected nodes' coordinate set (e) Case 4: coordinate set when all the nodes are anchors.

lower number of anchors, can produce more accurate, dimensionally scaled network. Though the latter was not our goal, it points to the possibility of obtaining even physically representative layout maps with appropriate selection of anchor nodes. Transformation to mitigate the deformations and effect of better anchor placement in topology maps are under investigation. The physical voids present in Fig. 8 are well preserved. Even though the map in Fig. 8 (e) was obtained using all the nodes as anchors its shape is deformed compared to Fig. 8 (b)-(d), but in terms of neighborhood preservation Fig. 8 (e) is better. For example, one of the L-shaped rooms in the building network (Fig. 8 (a)) is distorted in the topology maps of Fig. 8 (b)-(d). In Fig. 8 (e) the L-shape is deformed but neighborhood of that L-shaped room is preserved.

Topology Preservation Errors (Eq. (15)) for the four different topology maps of the four networks in Fig. 5-8 are presented in Table III. Note that the error in all the cases is less than 2%. The best performance in terms of Topology Preservation Error was achieved when all the nodes were selected as anchors for the networks in Fig. 5-8. Case 4 (Table II) acts as a lower bound for the Topology Preservation Error for each network.

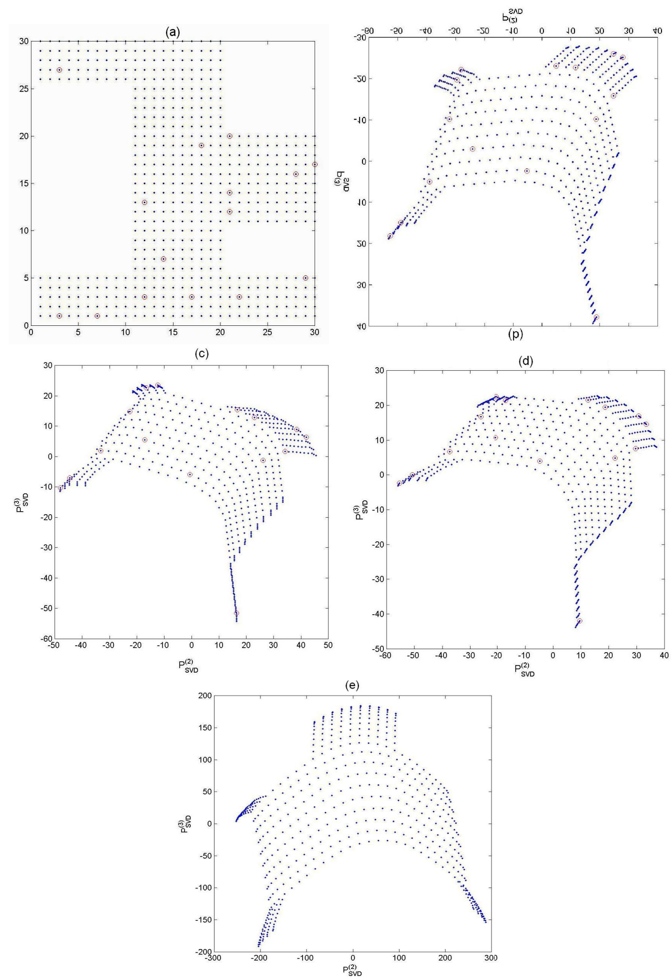

Figure 6. a) Odd shaped network; $[X_{SVD}, Y_{SVD}]$ is generated based on (b) Case 1; entire VC set,(c) Case 2: anchors' coordinate set, (d) Case 3: randomly selected nodes' coordinate set, (e) Case 4: coordinate set when all the nodes are anchors.

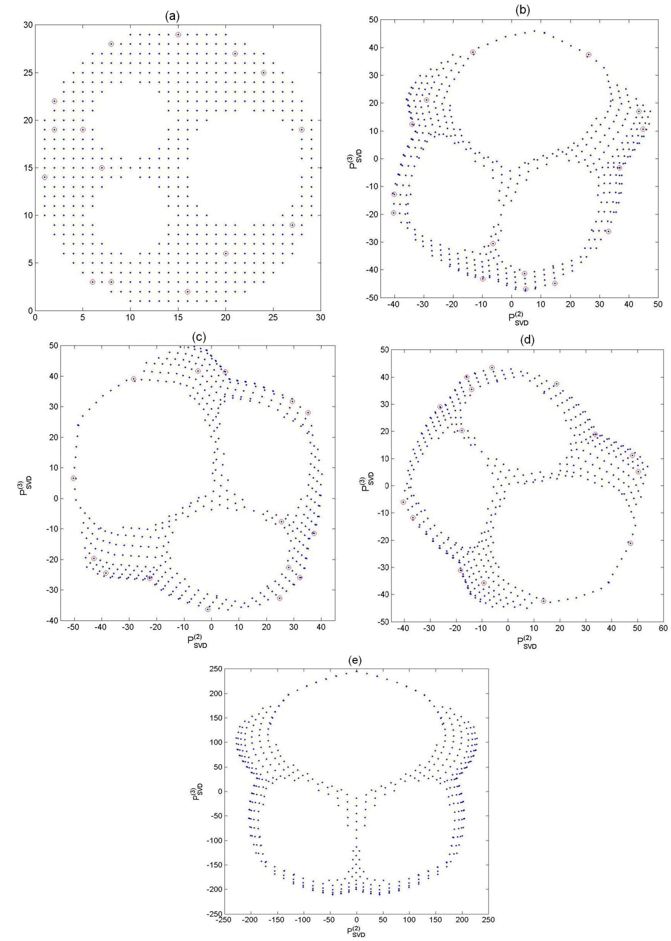

Figure 7. a) Circular network with three physical voids; $[X_{SVD}, Y_{SVD}]$ is generated based on (b) Case 1; entire VC set,(c) Case 2: anchors' coordinate set (d) Case 3: randomly selected nodes' coordinate set (e) Case 4: coordinate set when all the nodes are anchors.

Even though SVD based coordinate generation started with VC set where there is no directionality information, resultant topology map has directional information which can be used for routing in many ways for example to identify logical voids in VC routing, organized random routing and geographic routing in VS.

VI. Realizations of the Algorithm

In this section, we consider the realization of the algorithm in wireless sensor networks. As the major contribution of this paper is the technique described and evaluated above for generation of topology preserving map and the associated set of Cartesian coordinates, the details of realization are addressed only briefly. In static WSNs, the VC generation needs to be done very infrequently or perhaps only once during initialization. If a set of Cartesian coordinates is required, it needs to be done only once as well. Thus, the cost incurred in calculating Cartesian coordinates may be more than compensated by efficiency gains during long-term operation.

First, consider the case where the computation is done at a central node. There are many scenarios where a centralized implementation is feasible or even preferable. Consider a sensor network where the nodes are randomly deployed (e.g., dropped from a plane), and it is necessary to obtain a map of the sensor node deployment indicating geographic voids etc. In this case, each node may send information about its neighbors to a base or a central station. The adjacency matrix [20] of the network is formed based on the network node connectivity information, which can be gathered with the complexity of $O(N^2)$ where N is the number of nodes in the network. Now the above procedures (Section III) can be used to get a topology preserving map. If necessary, the map can be broadcast back to the individual nodes, together with the transformation matrix ($\underline{V}$ or $\underline{V_Q}$), an operation of complexity of $O(N^2)$. Generating coordinates at a central station avoids multiple flooding in the network [4][5][14][18].

A distributed implementation of the above may be achieved as follows. The anchor based VC generation is first carried out the traditional way, i.e., via flooding [1]. Following that, the Anchors broadcast their coordinates, which requires $O(MN)$ messages. Now that the array of all the anchors' coordinates is available at each node, each node i can generate $\underline{V_Q}$ $(Eq.\ (7))$ and evaluate its own $[x_{SVD,i}, y_{SVD,i}]$ locally by simply multiplying its own coordinate by $\underline{V_Q}$.

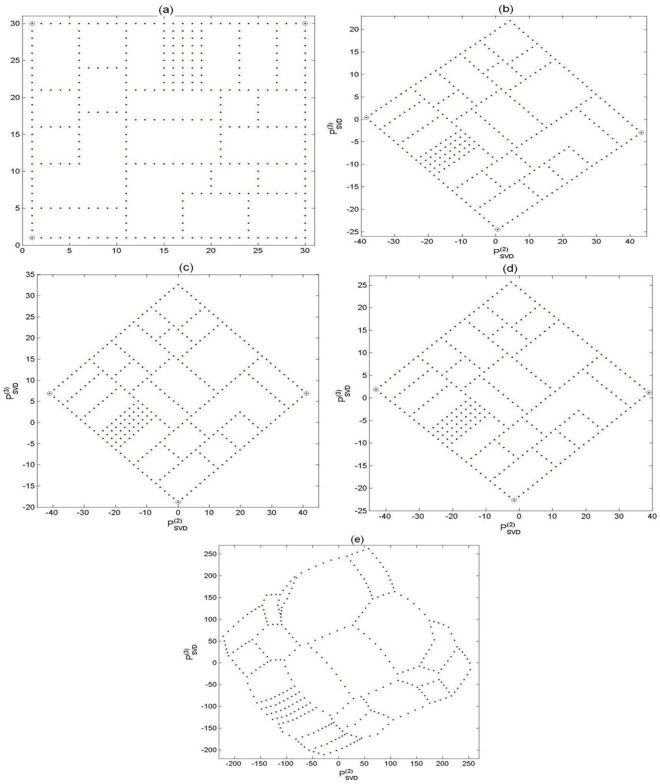

Figure 8. a) Network in a building; $[X_{SVD}, Y_{SVD}]$ is generated based on (b) Case 1; entire VC set,(c) Case 2: anchors' coordinate set (d) Case 3: randomly selected nodes' coordinate set (e) Case 4: coordinate set when all the nodes are anchors.

TABLE III. TOPOLOGY PRESERVATION ERROR FOR TOPOLOGY MAPS IN FIG. 5-8

	$\mathbf{E_{TP}}$ **(%)**			
Figure	**Case 1**	**Case 2**	**Case 3**	**Case 4**
Fig.5	0.9036	0.8346	0.2130	0
Fig.6	1.6777	1.5894	1.5011	1.4570
Fig.7	0.3605	1.0698	0.4884	0
Fig.8	0.1315	0.1315	0.1315	0.0376

VII. CONCLUSIONS AND FUTURE WORK

We presented a novel and a fundamental technique for generating a topology preserving map from virtual coordinates. As demonstrated, the transformation matrix for converting the logical coordinates to a set of approximate Cartesian coordinates can be obtained using the coordinates of a small set of nodes. Results show that a topology preservation error (E_{TP}) $\leq$ 2% is achievable.

Applications of topology preserving maps are diverse and vast: e.g., routing, localization, boundary node identification or improve anchor placement. Having VCs as well as approximate Cartesian coordinates (which can now be derived from VCs) can significantly enhance the routing mechanisms. Proposed SVD based approximated Cartesian coordinate generation method has many significant applications in WSNs as well as in general networks.

Methods to compensate for the compression of the map at the edges, and techniques that use derived Cartesian coordinates and the topology map to improve routability are under investigation.

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